**Question 1:**

1. If n < 0 then the program is not going to terminate because after every iteration of the function, n becomes smaller. Therefore it will never reach 0.
2. The proof is by induction on n, taking the induction hypothesis to be mult m n = m \* n.

Case (n = 0):

When n is zero the left-hand side will be mult m 0.

mult m 0

= {definition of the function mult}

0

The right-hand side will be m \* 0 = 0, therefore the claim holds.

Case (n = k + 1): Let us assume for n = k that mult m k = m \* k.

Let n = k + 1.

Left-hand side will be mult m (k + 1)

mult m (k + 1)

= {definition of the function mult}

m + mult m k

= {induction hypothesis}

m + m \* k

= {distributive property}

m \* (k + 1)

Right-hand side equals m \* (k + 1), so the proof is complete.

1. For every iteration of mult the second argument gets smaller by 1, so it takes n additions. The efficiency is O(n), which is fairly efficient.

**Question 2:**

1. The proof is by strong induction on n, taking the induction hypothesis to be

iter m n r = m \* n + r

Case (n = 0):

The left-hand side will be

iter m 0 r

= {definition of the function iter}

r

Right-hand side will be m \* 0 + r = r, so the hypothesis holds.

Case (n = 2k):

Let us assume that for n = k the hypothesis holds.

Let n be 2k.

iter m 2k r

= {definition of the function iter}

iter (m \* 2) k r

= {induction hypothesis}

2 \* m \* k + r

The right-hand side is m \* 2 \* k + r, which equals the left-hand side.

Case (n = 2k + 1):

Let us assume that for n = 2k the hypothesis holds.

Let n be 2k + 1.

iter m (2k + 1) r

= {definition of the function iter}

iter m 2k (r + m)

= {definition of the function iter}

m \* 2 \* k + r + m

= {distributive property}

m \* (2k + 1) + r

The right-hand side will be m \* (2k + 1) + r, which equals the left-hand side.

1. I have already proven that iter m n r = m \* n + r.

The definition of the function fastMult uses the function iter, therefore

fastMult m n

= {definition of fastMult}

iter m n 0

= {proof that iter m n r = m \* n + r}

m \* n

1. Every time n is even, it is being divided by 2, so it takes roughly iterations. The efficiency is O(log n), which is better than the previous multiplication function.

**Question 3:**

1. For the first seat there are 6 available people that could sit in it. For the second seat there will be 5 available people that could sit it in, and so on. Thus, there are 6! = 720 ways to seat those 6 people.
2. First let us look at the case in which the way they are seating is m f m f m f (m-male, f-female). For men and women there are 3! ways to seat them. Hence, 3!.3!. But the seating arrangement can be f m f m f m. So the total ways that they can sit so that genders alternate is 72.
3. Let us first see the case m f m f m f. The only arrangement in which they can sit without a couple sitting next to each other is 1 3 2 1 2 3 (people with same numbers are a couple). But 1, 2 and 3 have 6 permutations, so there are 6 ways in which they can sit in the arrangement m f m f m f. There is also another case which is f m f m f m. Then the total number of ways which satisfies the question are 12.

**Question 4:**

1. When a coin is tossed the chance of seeing heads is ½(the same is also true for tails). Because the three coin tosses are independent events, the chance of seeing heads, tails, heads in that order is (1/2)^3 = 1/8.
2. There are 4 possible outcomes from the three coin tosses:
3. 3 heads
4. 2 heads, 1 tails
5. 1 heads, 2 tails
6. 3 tails

After tossing the coin three times, the probability of seeing 3 heads are the same as seeing 3 tails, because the chance of flipping heads is the same as flipping tails. The same is true about 2) and 3). Therefore, the chance of seeing more heads than tails is ½.

1. Continuing the logic from b), there 100 possible outcomes: (99 heads), (98 heads, 1 tails), …, (99 tails)

The chance of seeing 99 heads is the same as seeing 99 tails. The chance of seeing 98 heads and 1 tails is the same as seeing 1 heads and 98 tails, and so on. Then the chance of getting more heads than tails is again ½.

Tutorial sheet from Tuan Nguen